

Impact on γ/ϕ_3 from CLEO-c Using CP -Tagged $D \rightarrow K_{S,L}\pi^+\pi^-$ Decays

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Precision determination of the CKM angle γ/ϕ_3 depends upon constraints on charm mixing amplitudes, measurements of doubly-Cabibbo suppressed amplitudes and relative phases, and studies of charm Dalitz plots tagged by flavor or CP eigenstates. In this note we describe the technique used at CLEO-c to constrain the $K_{S,L}\pi^+\pi^-$ model uncertainty, and its impact on γ/ϕ_3 measurements at B -factories presented at the Charm 2007 Workshop.

1. Introduction

Measurement of the CKM angle γ/ϕ_3 is challenging. Several methods have been proposed using $B^\mp \rightarrow DK^\mp$ decays; 1) the Gronau-London-Wyler (GLW) method [1] where the D decays to CP eigenstates 2) the Atwood-Dunietz-Soni (ADS) method [2] where the D decays to flavor eigenstates and 3) the Dalitz plot method [3, 5] where the D decays to a three-body final state. This latter method has been used recently by CLEO to measure the K^*K strong phase via the three-body decay $D^0 \rightarrow K^+K^-\pi^0$ [6]. Uncertainties due to charm contribute to each of these methods. The CLEO-c physics program includes a variety of charm measurements that will improve the determination of γ/ϕ_3 from the B -factory experiments, BaBar and Belle. The pertinent components of this program are improved constraints on charm mixing amplitudes - important for GLW, measurement of the relative strong phase δ between D^0 and \overline{D}^0 decay to $K^+\pi^-$ - important for ADS, and studies of charm Dalitz plots tagged by hadronic flavor or CP eigenstates. The total number of charm mesons accumulated at CLEO-c will be much smaller than the samples already accumulated by the B -factories. However, quantum correlations in the $D\overline{D}$ system from $\psi(3770)$ provides a unique laboratory in which to study charm.

The decay with the largest branching fraction relevant to the determination of γ/ϕ_3 $D^0 \rightarrow K_S\pi^+\pi^-$. Recently Babar [7] and Belle [8] have reported $\gamma = (92 \pm 41 \pm 11 \pm 12)^\circ$ and $\phi_3 = (53^{+15}_{-18} \pm 3 \pm 9)^\circ$, respectively, where the third error is the systematic error due to modeling of the Dalitz plot.

Both D^0 and \overline{D}^0 populate the Dalitz plots $K_S\pi^+\pi^-$, (as well as $\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$ and $K_S^0K^\pm\pi^\mp$) and so can be used in the determination of γ/ϕ_3 which exploit the interference between $b \rightarrow c\bar{u}s$ ($B^- \rightarrow D^0K^-$) and $b \rightarrow u\bar{c}s$ ($B^- \rightarrow \overline{D}^0K^-$) where the former process is real and the latter is proportional to $\sim e^{-i\gamma}$ [9]. Studying CP tagged Dalitz plots allows a model independent determination of the relative D^0 and \overline{D}^0 phase across the Dalitz plot. We describe this technique in the following sections.

2. Determining γ/ϕ_3 From B Decays

Our analysis follows the work outlined in [3], [5], and [4]. We consider the decay process $B^\pm \rightarrow DK^\pm$, followed by the three-body decay $D \rightarrow K_S\pi^+\pi^-$. Assuming no CP violation, we define the decay amplitudes for the D^0 and \overline{D}^0 to be

$$\begin{aligned} \mathcal{A}(D^0 \rightarrow K_S\pi^+\pi^-; x, y) &\equiv f_D(x, y) \\ \mathcal{A}(\overline{D}^0 \rightarrow K_S\pi^-\pi^+; x, y) &\equiv f_D(y, x). \end{aligned} \quad (1)$$

Sensitivity to the angle ϕ_3 comes from the interference of the neutral D mesons from $B^\pm \rightarrow DK^\pm$. Since the D meson is in a linear combination of flavor states, the amplitude for a $D^0 \rightarrow K_S\pi^+\pi^-$ event originating from a B decay is then

$$\mathcal{A}(B^- \rightarrow (K_S\pi^+\pi^-)_D K^-) \propto f_D + r_B e^{i\theta} f_{\overline{D}}, \quad (2)$$

up to an overall normalization. The angle θ is defined as $\theta_\pm \equiv \delta_B \pm \phi_3$. Here δ_B is the strong phase difference between color-suppressed and favored amplitudes, whilst r_B is the ratio between the color-suppressed to favored amplitudes. Theoretical estimates place r_B between 0.1-0.2 [11]. This has been confirmed by BaBar ($r_B = 0.12 \pm 0.08 \pm 0.03(\text{syst}) \pm 0.04(\text{model})$, [7]) and Belle ($r_B = 0.16 \pm 0.05 \pm 0.01(\text{syst}) \pm 0.05(\text{model})$, [8]).

The $D^0 \rightarrow K_S\pi^+\pi^-$ Dalitz plot is divided into $2N$ bins, symmetric under exchange of x and y . The bins are indexed from $-i$ to i , excluding zero, as in shown in Fig 1. The coordinate exchange $x \leftrightarrow y$ thus corresponds to the exchange of bins $i \leftrightarrow -i$. For simplicity we ignore the effects of efficiency and background in the Dalitz plot. The number of events in the i -th bin of the $K_S\pi^+\pi^-$ Dalitz plot from a D decay is then expressed as

$$K_i = A_D \int_{\mathcal{D}_i} |f_D(x, y)|^2 dx dy = A_D F_i. \quad (3)$$

The interference between the D^0 and \overline{D}^0 amplitudes is parametrized by the two quantities

$$c_i \equiv \frac{1}{\sqrt{F_i F_{-i}}} \int_{\mathcal{D}_i} \text{Re}[f_D(x, y) f_D^*(y, x)] dx dy \quad (4)$$

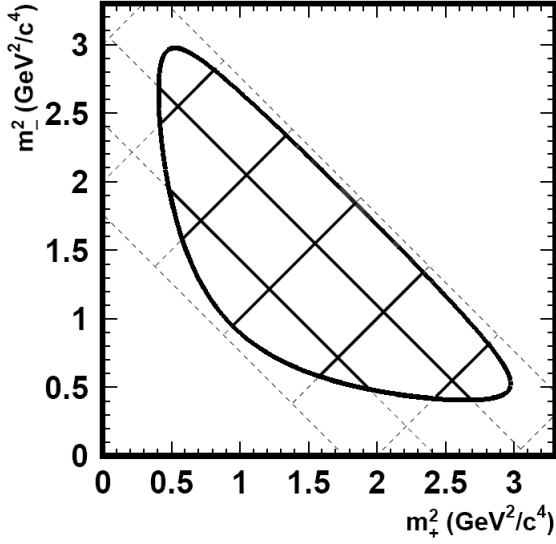


Figure 1: Binning of the $D^0 \rightarrow K_S \pi^+ \pi^-$ Dalitz plot.

and

$$s_i \equiv \frac{1}{\sqrt{F_i F_{-i}}} \int_{\mathcal{D}_i} \text{Im} [f_D(x, y) f_D^*(y, x)] dx dy, \quad (5)$$

where the integral is performed over a single bin. The number of events in the i -th bin of the $K_S \pi^+ \pi^-$ Dalitz plot from a B decay is then

$$N_i = K_i + r_B^2 K_{-i} - 2r_B \sqrt{K_i K_{-i}} (c_i \cos \theta_- - s_i \sin \theta_-), \quad (6)$$

again up to an overall normalization. It is important to note that c_i and s_i depend only on the D decay. These are the quantities that we measure using CLEO-c data. Although in principle they could be left as free parameters in a $D \rightarrow K_S \pi^+ \pi^-$ Dalitz plot analysis from B^\pm decays, their values can be more precisely determined from correlated D_{CP} decays produced at CLEO-c.

Thus, we can constrain θ_\pm , and in turn γ/ϕ_3 , if we know K_i , c_i , and s_i . The K_i can be easily determined using flavor-tagged $D^0 \rightarrow K_S \pi^+ \pi^-$ Dalitz plot. In the next section we show how the c_i can be obtained using binned, CP -tagged $D^0 \rightarrow K_S \pi^+ \pi^-$ Dalitz plots.

3. Measuring c_i From CP -Tagged D Decays

For D mesons that decay into a CP eigenstate, we write the initial state of the D as a linear combination of flavor eigenstates

$$f_{CP\pm}(x, y) = \frac{1}{\sqrt{2}} |f_D(x, y) \pm f_D(y, x)|. \quad (7)$$

In terms of this amplitude the number of events in the i -th bin of a CP -tagged Dalitz plot is

$$M_i^\pm = h_{CP\pm} \left(K_i \pm 2c_i \sqrt{K_i K_{-i}} + K_{-i} \right), \quad (8)$$

where $h_{CP\pm}$ is a normalization factor.

The expression given above for M_i^\pm can be used to measure c_i directly, even if only one type of CP tag is reconstructed. Care must be taken to use the corresponding value of $h_{CP\pm}$ as defined above. However, if samples of both CP parities are available we can combine the expressions for M_i^+ and M_i^- to get the following equation

$$c_i = \frac{1}{2} \frac{(M_i^- - M_i^+)}{(M_i^+ + M_i^-)} \frac{(K_i + K_{-i})}{\sqrt{K_i K_{-i}}}, \quad (9)$$

We thus have an expression for measuring c_i simply by counting events within the bins of flavor-tagged and CP -tagged Dalitz plots.

At CLEO-c we produce $D^0 \bar{D}^0$ pairs from the decay of a $\psi(3770)$ in a definite eigenstate of $C = -1$. Ignoring both the effects of CP violation, the double tag rate for final states $|1\rangle$ and $|2\rangle$ is given by

$$\Gamma(1, 2) = |A(1, 2)|^2 + (\text{mixing terms}), \quad (10)$$

where

$$A(1, 2) \equiv \langle 1 | D^0 \rangle \langle 2 | \bar{D}^0 \rangle - \langle 1 | \bar{D}^0 \rangle \langle 2 | D^0 \rangle \quad (11)$$

For the time being we ignore the effects of correlations and mixing in the $K\pi$ tagged Dalitz plot. This is not expected to make a significant difference for the $K\pi$ mode, as terms proportional to $r_{K\pi} \simeq 0.06$ and $r_{K\pi}^2$ are negligible.

3.1. Optimized Binning

Although the quantity s_i can only be measured using a $K_S \pi^+ \pi^-$ vs. $K_S \pi^+ \pi^-$ double Dalitz analysis [4], it can still be approximated from a single Dalitz plot if the binning is fine enough. If the bins are small enough that the phase difference and the amplitude remains constant across each bin, the strong phase parameters become $c_i = \cos(\delta_D)$, $s_i = \sin(\delta_D)$, so that the equality $s_i = \sqrt{1 - c_i^2}$ is true. It has been shown [3] that this equality holds for 200 or more bins, which is clearly not feasible for the number of D_{CP} tags produced at CLEO-c. In order to circumvent this problem, Bondar has proposed an alternate, model-dependent method for binning the $K_S \pi^+ \pi^-$ Dalitz plot [4]. The optimal choice depends on the $D^0 \rightarrow K_S \pi^+ \pi^-$ model. In this analysis we use the isobar model amplitude obtained from the most recent Belle ϕ_3 Dalitz analysis [8].

From the consideration above it is clear that a good approximation to the optimal binning is the one obtained from the uniform division of the strong phase

difference δ_D . We thus take the definition of i -th bin to be

$$2\pi(i - 1/2)/N \leq \delta_D(x, y) < 2\pi(i + 1/2)/N. \quad (12)$$

An example of such a binning with $N = 8$ is shown in Fig. 2.

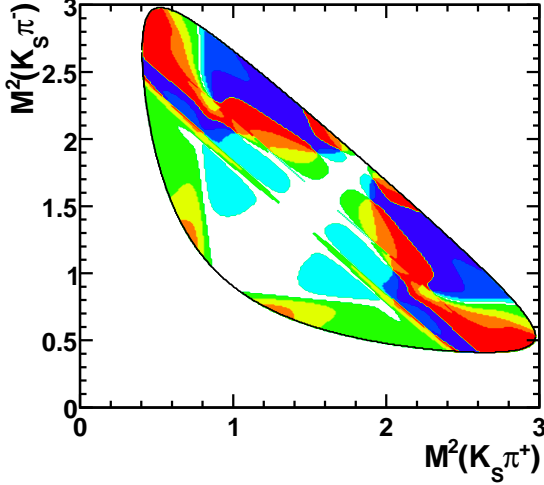


Figure 2: Divisions of the $D^0 \rightarrow K_S \pi^+ \pi^-$ Dalitz plot with uniform binning of $\Delta\delta_D$ strong phase difference with $N = 8$.

4. Event Selection

4.1. Double-Tagged $D \rightarrow K_S \pi^+ \pi^-$ Events

This analysis uses a combination of two-body CP and flavor tags. Since the neutral D mesons are produced at $\psi'(3770)$ threshold they are correlated in a $C = -1$ state. If mixing is ignored we can determine whether the parent particle was a D^0 or \overline{D}^0 , up to DCS contributions. Similarly, if CP violation is ignored, then the D mesons must be in eigenstates of opposite CP [10].

To determine the flavor of the D meson, we tag $D^0 \rightarrow K_S \pi^+ \pi^-$ events with the two-body $\overline{D}^0 \rightarrow K^+ \pi^-$ mode.¹ We use the two CP -even tags $K^+ K^-$ and $\pi^+ \pi^-$, and the two CP -odd tags $K_S \pi^0$ and $K_S \eta$.

We introduce two quantities that are reconstructed on both sides of a double-tagged decay. The beam-constrained mass is defined as $M_{bc} \equiv \sqrt{E_b^2 - p_D^2}$,

where E_b is the beam energy and p_D^2 is the square of the reconstructed 3-momentum of the D meson. We require that the beam-constrained mass of the reconstructed candidate is within 3σ of the nominal D mass, which corresponds to a selection criteria of $1.8603 \leq M_{bc} \leq 1.8687$ GeV. The other quantity is the energy difference between the beam and the reconstructed D , defined as $\Delta E \equiv E_{beam} - E_D$. We apply a selection criteria of $|\Delta E| \leq 30$ MeV to all $D^0 \rightarrow K_S \pi^+ \pi^-$ candidates.

Additional selection criteria are placed on the daughter particles to ensure basic track quality. For example, we select pion track momenta between $0.05 \leq p \leq 2.0$ GeV. Both signal and tagging modes containing a K_S are selected to be within 3σ of the K_S mass, which corresponds to ± 7.5 MeV from the central K_S mass value of 497.6 MeV.

We only reconstruct K_S particles that decay through the $\pi^+ \pi^-$ channel; we do not attempt to reconstruct $K_S \rightarrow \pi^0 \pi^0$. Fake K_S candidates can be misreconstructed from combinatoric $\pi^+ \pi^-$ pairs. To suppress these events we apply a selection criteria on the flight significance $f_s \geq 0$ to our K_S candidates. Additionally, we require that the π^0 mass falls within 3σ of its nominal value.

4.2. Double-Tagged $D \rightarrow K_L \pi^+ \pi^-$ Events

For $D^0 \rightarrow K_L \pi^+ \pi^-$ decays we require the same selection criteria on charged pions and π^0 candidates as those described for $D^0 \rightarrow K_S \pi^+ \pi^-$ decays. However, because of the large flight distance of the K_L , the $K_L \pi^+ \pi^-$ signal is reconstructed using a missing mass technique. We require the signal side to have exactly two charged tracks. We also apply π^0 , η , and K_S vetoes. Using the measured momentum of the tagged D , we compute the missing momentum and energy on the signal side. We require that the missing mass squared satisfies the condition $0.21 \leq m^2 \leq 0.29$ GeV². The background for $D^0 \rightarrow K_L \pi^+ \pi^-$ mode is approximately 5%.

4.3. Double-Tagged $K_L \pi^0$ vs. $K_S \pi^+ \pi^-$

We can increase our statistics by reconstructing $D^0 \rightarrow K_S \pi^+ \pi^-$ events tagged with the CP -even mode $K_L \pi^0$. We require zero tracks and exactly one π^0 candidate on the tag side. We veto events containing η candidates, and impose similar criteria on the K_L missing mass as described above.

The final yields for all tag modes are summarized in Table I

¹The inclusion of charge-conjugate modes is implied throughout our analysis.

Table I Yields for CP-tagged $K_S\pi^+\pi^-$ and $K_L\pi^+\pi^-$ in 398 pb^{-1} data, by tag mode.

Tag Mode	$K_S\pi^+\pi^-$	$K_L\pi^+\pi^-$
K^+K^-	61	194
$\pi^+\pi^-$	33	90
$K_S\pi^0$	108	263
$K_S\eta$	29	21
$K_L\pi^0$	190	-

5. Combining $K_S\pi^+\pi^-$ and $K_L\pi^+\pi^-$

The tagged $K_L\pi^+\pi^-$ Dalitz plots are included to increase the statistical accuracy of this analysis. However, if we naively combine the Dalitz plots with K_S and K_L we will find our measurement of c_i to be biased. We must first account for the phenomenological differences between the $K_S\pi^+\pi^-$ and $K_L\pi^+\pi^-$ models.

Since the K_S and K_L mesons are of opposite CP , the doubly-Cabibbo suppressed amplitudes in each Dalitz plot will contribute with opposite signs. We can see this by inspecting the D^0 decay amplitude for each Dalitz plot

$$\begin{aligned} \mathcal{A}(K_S\pi\pi) &= \frac{1}{\sqrt{2}} \left[\mathcal{A}(K^0\pi\pi) + \mathcal{A}(\overline{K}^0\pi\pi) \right] \\ \mathcal{A}(K_L\pi\pi) &= \frac{1}{\sqrt{2}} \left[\mathcal{A}(K^0\pi\pi) - \mathcal{A}(\overline{K}^0\pi\pi) \right] \end{aligned} \quad (13)$$

$$(14)$$

The effect of this relative minus sign is to introduce a 180° phase for all DCS K^* resonances in the $K_L\pi^+\pi^-$ model. We can use U -spin symmetry to relate the amplitudes for resonances of definite CP eigenvalue. We find that these states acquire a factor of $r_K e^{i\delta_K} \simeq -\tan\theta_C$, where θ_C is the Cabibbo angle.

In our study we multiply all DCS amplitudes in the $K_L\pi^+\pi^-$ model by -1. From this “base” model we fix $r_K = 0.06$ for each CP eigenstate, then vary the phase δ_K between 0 and 2π . For each bin we then find the largest resulting deviation in c_i , and report this value as the systematic uncertainty in the $K_L\pi^+\pi^-$ model.

To better understand the difference between the $K_S\pi^+\pi^-$ and $K_L\pi^+\pi^-$ models, we compare the numerically calculated values of c_i in each Dalitz plot. We find that the value for c_i is systematically larger in each bin for $K_L\pi^+\pi^-$. In Fig. 3 we can see that the difference is significantly larger than the systematic uncertainty in our $K_L\pi^+\pi^-$ model.

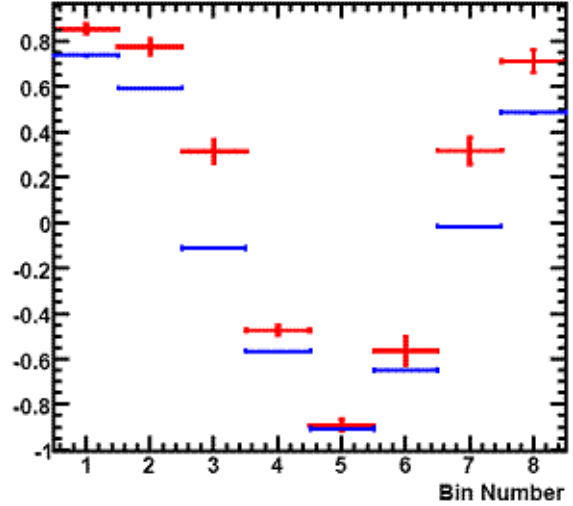


Figure 3: Values for c_i numerically determined from our model. $K_L\pi^+\pi^-$ values are in red, $K_S\pi^+\pi^-$ in blue. In each bin the values for c_i are systematically larger for $K_L\pi^+\pi^-$. The error bars represent the uncertainties in the $K_L\pi^+\pi^-$ model parametrization.

6. Results

We report the difference in c_i between $K_S\pi^+\pi^-$ and $K_L\pi^+\pi^-$ as measured in 398 pb^{-1} of data. In Fig. 4 we compare the c_i differences calculated from our model and measured from data. The error bars

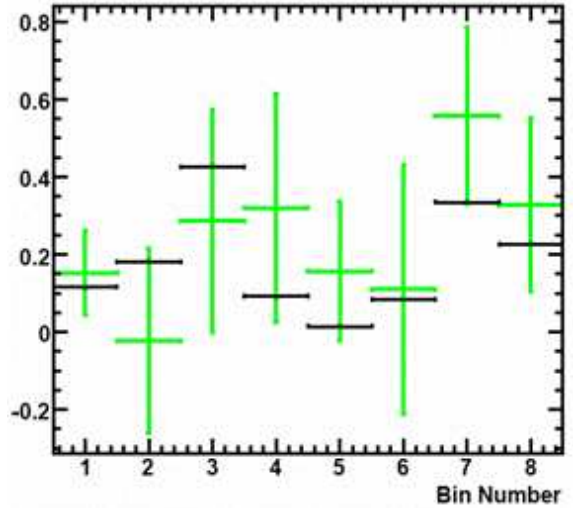


Figure 4: The difference in $K_S\pi\pi$ and $K_L\pi\pi$ values of c_i , numerically determined from our model (black) and measured in 398 pb^{-1} of data (green). The green error bars represent the combined statistical and model uncertainty.

in this figure represent both statistical and model uncertainty combined. With a reasonable understanding of the c_i between the $K_S\pi^+\pi^-$ and $K_L\pi^+\pi^-$ Dalitz

plots, we can estimate the final precision with which we expect to measure c_i once 750 pb^{-1} of data is available. The values of c_i from our study are once again plotted in Fig. 5, but here the error bars represent the statistical uncertainty obtained from 398 pb^{-1} of data scaled up to 750 pb^{-1} .

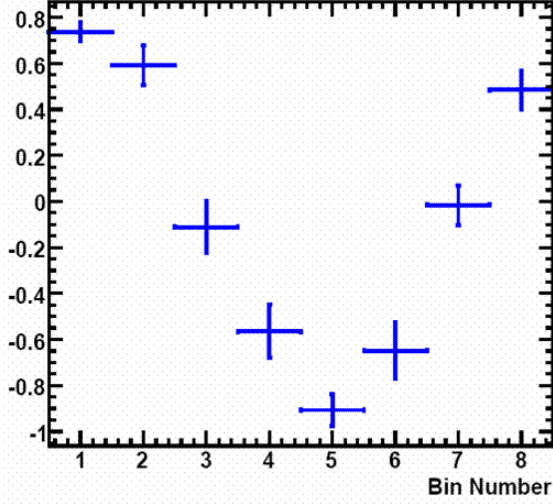


Figure 5: The central values of c_i are computed from our model with expected sensitivity from 750 pb^{-1} of data. The error bars are determined by scaling the statistical uncertainty obtained from 398 pb^{-1} of data, then combining the $K_L\pi^+\pi^-$ model uncertainty.

We expect good sensitivity to the measurement of c_i with the entire CLEO-c data. This measurement can reduce the model uncertainty on γ/ϕ_3 to a precision of about 4° [3].

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